chapter: 3 POLYNOMIALS

section: 3

21. check whether -2 and 3 are the zeroes of polynomial $p(x) = x^2 - x - 6$?

sol:- given $p(x) = x^2 - x - 6$.

$p(3) = (3)^2 - 3 - 6 = 9 - 3 - 6 = 9 - 9 = 0$

$p(-2) = (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 6 - 6 = 0$

Hence 3 & -2 are the zeroes of the polynomial.

22. why are $1/4$ and -1 zeroes of the polynomial $p(x) = 4x^2 + 3x - 1$?

sol:- given $p(x) = 4x^2 + 3x - 1$.

$p(1/4) = 4(1/4)^2 + 3(1/4) - 1$

$= \cancel{4}\left(\frac{1}{\cancel{16}_4}\right) + 3(1/4) - 1$

$= \frac{1}{4} + \frac{3}{4} - \frac{1}{1} = \frac{1+3-4}{4} = 0$.

$p(-1) = 4(-1)^2 + 3(-1) - 1$

$= 4 - 3 - 1 = 4 - 4 = 0$

$\therefore$ $1/4$ and -1 are the zeroes of the polynomial.

23. Let $p(x) = x^2 - 4x + 3$; find the value of $p(0), p(1), p(2), p(3)$ and obtain zeroes of the polynomial $p(x)$?

sol:- given $p(x) = x^2 - 4x + 3$.

$p(0) = 0^2 - 4(0) + 3 = 3$.

$p(1) = (1)^2 - 4(1) + 3 = 1 - 4 + 3 = 4 - 4 = 0$

$p(2) = (2)^2 - 4(2) + 3 = 4 - 8 + 3 = 7 - 8 = -1$

$p(3) = (3)^2 - 4(3) + 3 = 9 - 12 + 3 = 12 - 12 = 0$

$p(x) = x^2 - 4x + 3$. 　 3 : -1, -3

$= x^2 - x - 3x + 3$

$= x(x-1) - 3(x-1)$

$= (x-1)(x-3)$.

$p(x) = 0 \Rightarrow (x-1)(x-3) = 0$

$\Rightarrow x = 1, 3$.

$\therefore$ zeroes of the polynomial $= 1$ & 3.

24. Find the zeroes of the polynomial $p(x) = x^2 + 7x + 10$ and verify the relationship b/w zeroes and coefficients?

sol:- given $p(x) = x^2 + 7x + 10$

$= x^2 + 5x + 2x + 10$

$= x(x+5) + 2(x+5)$

$= (x+5)(x+2)$

then the zeroes are: -5 & -2.

Verify:-

Sum of zeroes: $(-2)+(-5) = -7 = -\frac{7}{1} = \frac{-(\text{coeff of } x)}{\text{coefficient of } x^2}$

product of zeroes $= (-2)(-5) = 10 = \frac{10}{1} = \frac{\text{constant}}{\text{coefficient of } x^2}$

25, find The zeroes of The polynomial $p(x) = x^2-2x-8$ and verify The relationship b/w The zeroes and coefficients.

sol: given $p(x) = x^2-2x-8$.

$= x^2-4x+2x-8$

$= x(x-4)+2(x-4)$

$= (x-4)(x+2)$

8: $-4+2$; 1 8

$p(x) = 0$

$(x-4)(x+2) = 0$

$\Rightarrow x = 4$ or -2.

$\therefore$ The zeroes of x^2-2x-8 are 4 or -2.

Verify:-

sum of zeroes: $4+(-2) = 2 = \frac{2}{1} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

product of zeroes: $4(-2) = -8 = \frac{-8}{1} = \frac{(\text{constant term})}{\text{coefficient of } x^2}$

26, Find a Quadratic polynomial if The zeroes of it are 2 and $-\frac{1}{3}$ respectively

sol: Let The Quadratic polynomial be ax^2+bx+c; $a \neq 0$ and its zeroes be α and β.

Here $\alpha = 2$ and $\beta = -\frac{1}{3}$.

Sum of zeroes: $(\alpha+\beta) = 2+\left(-\frac{1}{3}\right) = \frac{6-1}{3} = \frac{5}{3}$

product of zeroes: $\alpha\beta = 2\left(-\frac{1}{3}\right) = -\frac{2}{3}$.

$\therefore$ The Quadratic polynomial ax^2+bx+c is $k[x^2-(\alpha+\beta)x+\alpha\beta]$; "k" is constant

$= k\left[x^2-\frac{5}{3}x-\frac{2}{3}\right]$; $k \in \mathbb{R}$.

when $k = 3$; The Quadratic polynomial is $3x^2-5x-2$.

$\therefore$ The required polynomial is $3x^2-5x-2$.

27. Divide $3x^3+x^2+2x+5$ by $1+2x+x^2$?

Sol:

$$x^2+2x+1 \overline{)\, 3x^3+x^2+2x+5\,} (3x-5$$
$$\underline{3x^3 + 6x^2 + 3x}$$
$$-5x^2-x+5$$
$$\underline{-5x^2 - 10x - 5}$$
$$9x+10$$

$\frac{3x^3}{x^2} = 3x$

$3x(x^2+2x+1) = 3x^3+6x^2+3x$

$\frac{-5x^2}{x^2} = -5$

$-5(x^2+2x+1) = -5x^2-10x-5$

28 Find a Quadratic Polynomial; The sum and product of whose zeroes are $1/4$ and -1 respectively.

Sol: Let the Quadratic polynomial be ax^2+bx+c; $a \neq 0$ and its zeroes are α & β

~~$\alpha = 1/4$; $\beta = -1$~~

We have; $\alpha+\beta = 1/4 = \frac{-b}{a}$

$\alpha\beta = -1 = \frac{c}{a}$

If we take; $a = 4$ Then $b = -1$; $c = -4$.

So The Quadratic polynomial; which given condition satisfies is $4x^2-x-4$.

Section: 4

29 Verify That $1, -1, -3$ are The zeroes of The Polynomial x^3+3x^2-x-3 and Then verify the relationship b/w The zeroes and coefficients?

Sol: comparing The given polynomial with ax^3+bx^2+cx+d; we get

$a = 1$; $b = 3$; $c = -1$; $d = -3$.

$P(1) = (1)^3 + 3(1)^2 - 1 - 3 = 1+3-1-3 = 0$.

$P(-1) = (-1)^3 + 3(-1)^2 - (-1) - 3 = -1+3+1-3 = 0$.

$P(-3) = (-3)^3 + 3(-3)^2 - (-3) - 3 = -27+27+3-3 = 0$.

Therefore $1, -1, -3$ are The zeroes of x^3+3x^2-x-3.

So we take $\alpha = 1$; $\beta = -1$; $\gamma = -3$

Now, $\alpha+\beta+\gamma = 1 + (-1) + (-3) = -3 = \frac{-3}{1} = \frac{-b}{a}$

$\alpha\beta+\beta\gamma+\gamma\alpha = 1(-1) + (-1)(-3) + (-3)(1)$
$= -1 + \cancel{3} - \cancel{3} = -1 = -\frac{1}{1} = \frac{c}{a}$

$\alpha\beta\gamma = (1)(-1)(-3) = \frac{3}{1} = \frac{-d}{a}$

$\therefore$ $1, -1, -3$ are The zeroes of The polynomial.

(OR)

ii) verify that $3, -1, -\frac{1}{3}$ are the zeroes of the polynomial $3x^3-5x^2-11x-3$ and verify the relationship b/w zeroes and coefficients.

comparing given polynomial with ax^3+bx^2+cx+d; we get

$a=3$; $b=-5$; $c=-11$; $d=-3$.

$P(3) = 3(3)^3-5(3)^2-11(3)-3 = 81-45-33-3 = 0$

$P(-1) = 3(-1)^3-5(-1)^2-11(-1)-3 = -3-5+11-3 = 0$

$P(-\frac{1}{3}) = 3(-\frac{1}{3})^3-5(-\frac{1}{3})^2-11(-\frac{1}{3})-3 = -\frac{1}{9}-\frac{5}{9}+\frac{11}{3}-3 = -\frac{2}{3}+\frac{2}{3} = 0$.

Therefore $3, -1, -\frac{1}{3}$ are the zeroes of $3x^3-5x^2-11x-3$.

so we take $\alpha=3$, $\beta=-1$; $\gamma=-\frac{1}{3}$.

Now $\alpha+\beta+\gamma = 3+(-1)+(-\frac{1}{3}) = 2-\frac{1}{3} = \frac{5}{3}$

$\alpha\beta+\beta\gamma+\gamma\alpha = 3(-1)+(-1)(-\frac{1}{3})+(-\frac{1}{3})(3) = -3+\frac{1}{3}-1 = \frac{-11}{3} = \frac{c}{a}$

$\alpha\beta\gamma = 3(-1)(-\frac{1}{3}) = 1 = \frac{-(-3)}{3} = \frac{-d}{a}$

30) find all zeroes of $2x^4-3x^3-3x^2+6x-2$, if you know that two of its zeroes are $\pm\sqrt{2}$.

since two of the zeroes are $\sqrt{2}$ and $-\sqrt{2}$. Therefore we can divide by $(x-\sqrt{2})(x+\sqrt{2}) = x^2-2$.

$$x^2-2)\ 2x^4-3x^3-3x^2+6x-2\ (2x^2-3x+1$$
$$\underline{\ominus 2x^4 \quad \oplus 4x^2}$$
$$-3x^3+x^2+6x-2$$
$$\underline{\ominus -3x^3 \quad \pm 6x}$$
$$x^2-2$$
$$\underline{x^2-2}$$
$$0$$

$\frac{2x^4}{x^2} = 2x^2$

$2x^2(x^2-2) = 2x^4-4x^2$

$\frac{-3x^3}{x^2} = -3x$

$-3x(x^2-2) = -3x^3+6x$

$\frac{x^2}{x^2} = 1$.

so; $2x^4-3x^3-3x^2+6x-2 = (x^2-2)(2x^2-3x+1)$.

on factorise $2x^2-3x+1$

$2x^2-2x-x+1$

$2x(x-1)-1(x-1)$

$(x-1)(2x-1)$

the zeroes are $1, \frac{1}{2}$.

Therefore, the zeroes of given polynomial are: $\sqrt{2}, -\sqrt{2}, 1, \frac{1}{2}$.

chapter: 3 Polynomials

section: 4

30 (b) Find ~~what value of~~ ~~q~~; The ~~equations~~ ~~all~~ zeroes of $3x^4+6x^3-2x^2-10x-5$ if you know that two of its zeroes are $\sqrt{5/3}$ & $-\sqrt{5/3}$.

sol: Since two of its zeroes are $\sqrt{5/3}$ & $-\sqrt{5/3}$.

Therefore we can divide by $(x-\sqrt{5/3})(x+\sqrt{5/3}) = x^2-5/3$.

$$x^2-\frac{5}{3}\,\big)\; 3x^4+6x^3-2x^2-10x-5\;\big(\,3x^2+6x+3$$

$$\begin{array}{l} \cancel{3x^4} \qquad \ominus 5x^2 \\ \hline 6x^3+3x^2-10x-5 \\ 6x^3 \qquad\quad -10x \\ \hline 3x^2 \quad -5 \\ 3x^2 \quad -5 \\ \hline 0 \end{array}$$

$\frac{3x^4}{x^2} = 3x^2$

$3x^2(x^2-5/3) = 3x^4-5x^2$

$\frac{6x^3}{x^2} = 6x$

$6x(x^2-5/3) = 6x^3-10x$

$\frac{3x^2}{x^2} = 3$

$3(x^2-5/3) = 3x^2-5$.

So $3x^4+6x^3-2x^2-10x-5 = (x^2-5/3)(3x^2+6x+3)$.

on dividing $3x^2+6x+3$ by 3 to get x^2+2x+1.

Then we factorise as $(x+1)(x+1)$; so its zeroes are $-1, -1$.

∴ The zeroes of given polynomial are $-1, -1, \sqrt{5/3}, -\sqrt{5/3}$.

32 (i) Divide $3x^2-x^3-3x+5$ by $x-1-x^2$; and verify the division algorithm?

sol: dividend $= -x^3+3x^2-3x+5$

divisor $= -x^2+x-1$

$$-x^2+x-1\,\big)\; -x^3+3x^2-3x+5\;\big(\,x-2$$

$$\begin{array}{l} \ominus x^3 \oplus x^2 \ominus x \\ \hline 2x^2 \quad -2x+5 \\ 2x^2 \quad -2x+2 \\ \ominus \qquad \oplus \quad \ominus \\ \hline 3 \end{array}$$

We know That, $p(x) = g(x)\cdot q(x) + r(x)$

$= (-x^2+x-1)(x-2)+3$

$= -x^3+x^2-x+2x^2-2x+2+3$

$p(x) = -x^3+3x^2-3x+5$

∴ Division algorithm is verified.

32 (ii) Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeroes are $+2, -7, -14$ respectively.

Sol: Let the cubic polynomial be ax^3+bx^2+cx+d; $a \neq 0$.

(i) Sum of zeroes:-

given sum of zeroes $= 2$.

$\Rightarrow \frac{-b}{a} = 2 = \frac{2}{1}$

Assuming; $a=1$ and $b=-2$.

(ii) sum of product of zeroes:-

given sum of product of zeroes $= -7$.

$\Rightarrow \frac{c}{a} = -7 = \frac{-7}{1}$

Assuming; $a=1$ and $c=-7$.

(iii) product of zeroes:-

given product of zeroes $= -14$.

$\Rightarrow \frac{-d}{a} = -\frac{14}{1}$

$\Rightarrow d = 14$ & $a=1$.

Thus; $a=1$; $b=-2$; $c=-7$; $d=14$.

$\therefore$ Hence required polynomial; $p(x) = ax^3+bx^2+cx+d$

$\Rightarrow p(x) = x^3-2x^2-7x+14$

33 (i) On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x-2$ and $-2x+4$, respectively, find $g(x)$?

Sol: Dividend $= x^3-3x^2+x+2$.

Quotient $= x-2$

Remainder $= -2x+4$.

W.K.T; Division algorithm in polynomials is;

$\boxed{p(x) = g(x) \cdot q(x) + r(x)}$

$\Rightarrow x^3-3x^2+x+2 = g(x) \cdot (x-2) + (-2x+4)$

$\Rightarrow x^3-3x^2+x+2-(-2x+4) = g(x) \cdot (x-2)$

$\Rightarrow x^3-3x^2+x+2+2x-4 = g(x) \cdot (x-2)$.

$\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x)(x-2)$

$\Rightarrow g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{(x-2)}$

NOW;

$$
\begin{array}{r|l}
x-2 & x^3 - 3x^2 + 3x - 2 \;(x^2 - x + 1 \\
 & \underline{x^3 - 2x^2} \\
 & -x^2 + 3x - 2 \\
 & \underline{-x^2 + 2x} \\
 & x - 2 \\
 & \underline{x - 2} \\
 & 0
\end{array}
$$

$\dfrac{x^3}{x} = x^2$

$x^2(x-2) = x^3 - 2x^2$

$\dfrac{-x^2}{x} = -x$

$-x(x-2) = -x^2 + 2x$

$\dfrac{x}{x} = 1$

$1(x-2) = x-2$

so; $\dfrac{x^3 - 3x^2 + 3x - 2}{x-2} = x^2 - x + 1$

$\therefore g(x) = x^2 - x + 1$

33 ii, Give examples of polynomials $p(x), g(x), q(x)$ and $r(x)$, which satisfy the division algorithm and (i) $\deg p(x) = \deg q(x)$ (ii) $\deg q(x) = \deg r(x)$ iii, $\deg r(x) = 0$?

Sol:-

i, $\deg p(x) = \deg q(x)$

Let $p(x) = 5x^2 - 5x + 10$; $g(x) = 5$

$$
\begin{array}{r|l}
5 & 5x^2 - 5x + 10 \;(x^2 - x + 2 \\
 & \underline{5x^2} \\
 & -5x \\
 & \underline{-5x} \\
 & 10 \\
 & \underline{10} \\
 & 0
\end{array}
$$

$p(x) = g(x) \cdot q(x) + r(x)$

$\deg p(x) = \deg q(x) = 2$

ii, $\deg q(x) = \deg r(x)$

Let $p(x) = 4x^3 + x^2 + 3x + 6$

$g(x) = x^2 + 3x + 1$

NOW,

$$x^2+3x+1 \overline{\smash{)}\,4\cancel{x^3}+x^2+3x+6\,}\,(\,4x-11$$

$$\cancel{4x^3}+12x^2+4x$$

$$-11\cancel{x^2}-x+6$$

$$\ominus 11\cancel{x^2} \ominus 33x \ominus 11$$

$$32x+17$$

Here $q(x) = 4x-11$; $r(x) = 32x+17$.

$\therefore$ $\deg q(x) = \deg r(x)$.

iii, $\deg r(x) = 0$

Let $p(x) = 7x^3 - 42x + 53$.

$g(x) = x^3 - 6x + 7$.

NOW;

$$x^3-6x+7 \overline{\smash{)}\,7\cancel{x^3}-\cancel{42}x+53\,}\,(\,7$$

$$\cancel{7x^3} \ominus \cancel{42}x \ominus 49$$

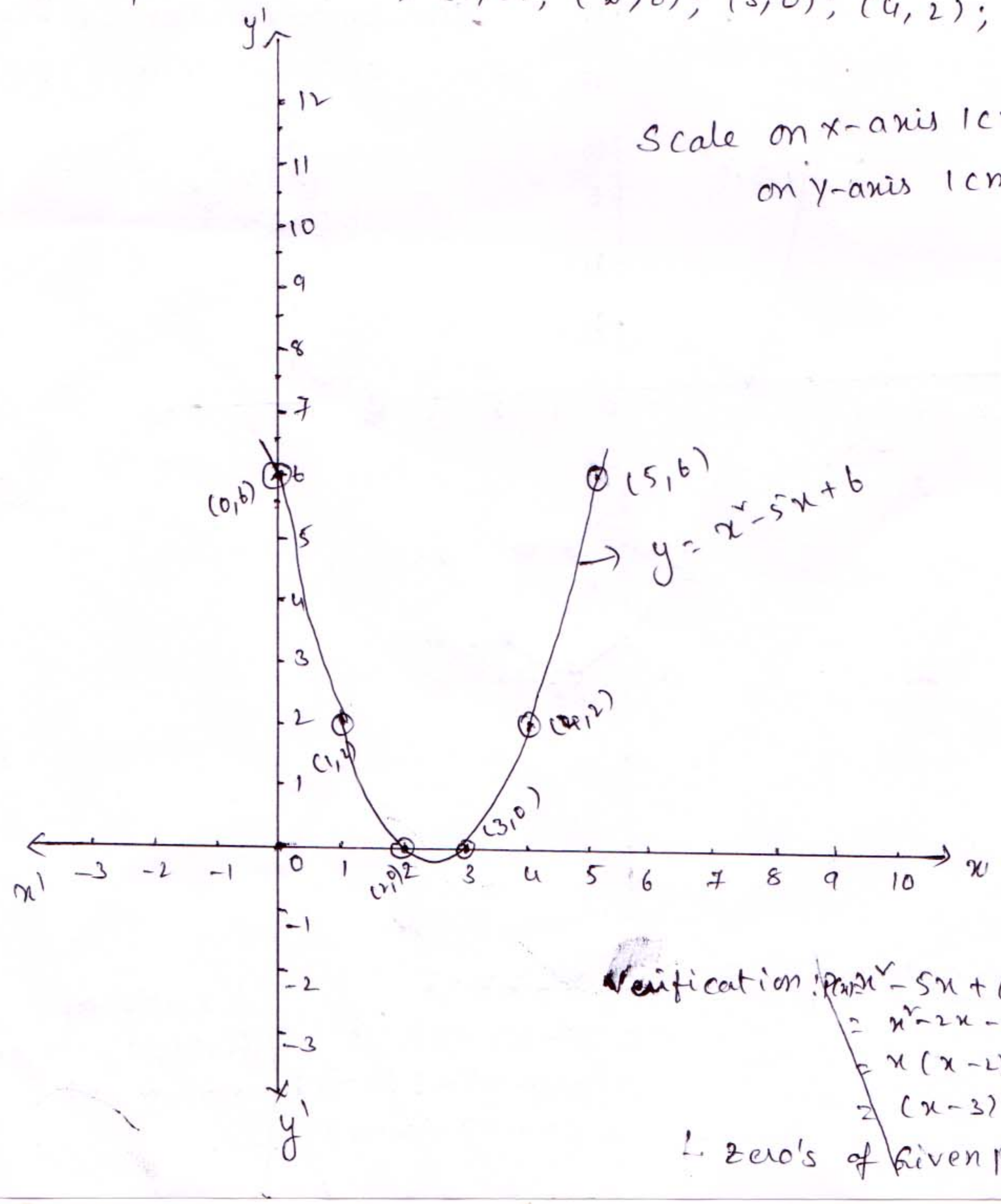
$$4$$

$\therefore \deg r(x) = 0$.

31. Given $y = x^2 - 5x + 6$

x	0	1	2	3	4	5
x^2	0	1	4	9	16	25
$-5x$	0	-5	-10	-15	-20	$-\cancel{30}\,25$
6	6	6	6	6	6	6
y	6	2	0	0	2	6

order pairs $(0,6)$; $(1,2)$; $(2,0)$; $(3,0)$; $(4,2)$; $(5,6)$



Scale on x-axis 1cm = 1unit
on y-axis 1cm = 1unit

Verification: $p(x) = x^2 - 5x + 6$
$= x^2 - 2x - 3x + 6$
$= x(x-2) - 3(x-2)$
$= (x-3)(x-2)$

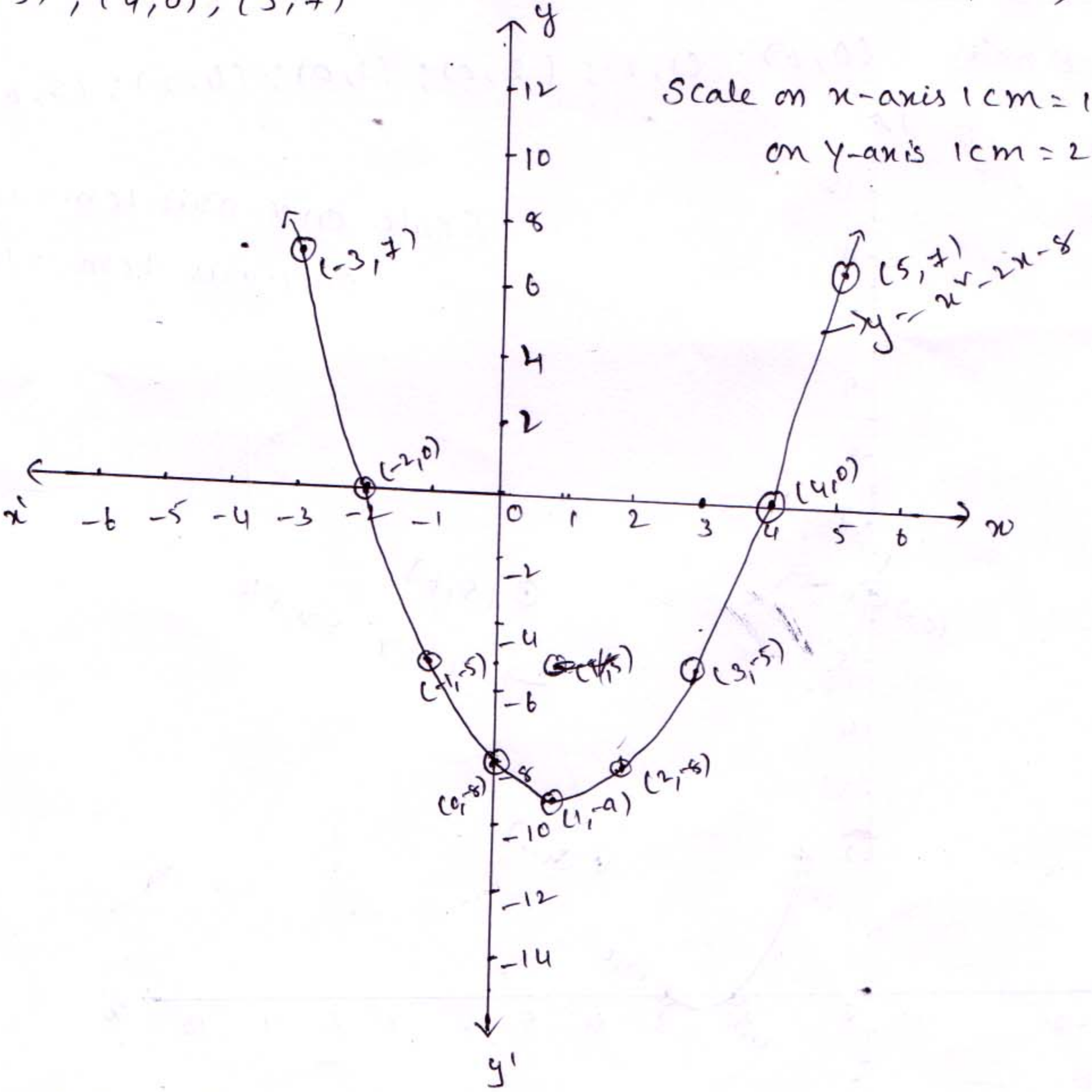
$\therefore$ zero's of given polynomial are 2, 3

b) Given $y = x^2 - 2x - 8$

x	-3	-2	-1	0	1	2	3	4	5
x^2	9	4	+1	0	1	4	9	16	25
$-2x$	6	4	2	0	-2	-4	-6	-8	-10
-8	-8	-8	-8	-8	-8	-8	-8	-8	-8
y	7	0	-5	-8	-9	-8	-5	0	7

Order pairs are (-3, 7); (-2, 0); (-1, -5); (0, -8); (1, -9); (2, -8) (3, -5); (4, 0); (5, 7)

Scale on x-axis 1 cm = 1 unit
on y-axis 1 cm = 2 units.



Verification:- $P(x) = x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x-4) + 2(x-4)$

$= (x-4)(x+2)$

∴ zeros of Given polynomial are 4, -2